Relativity Questions

Part 1: The Postulates of Special Relativity

1. What one measurement will two observers in relative motion *always* agree upon?
2. Some of the distant stars, called quasars, are receding from us at half the speed of light (or greater). What is the speed of light we receive from these quasars?
3. In a laboratory frame of reference, an observer notes that Newton's second law is valid. Show that it is also valid for an observer moving at a constant speed relative to the laboratory frame. I.e. show that $F = ma = m\frac{\Delta v}{\Delta t}$ in the new reference frame given it did in the laboratory frame. Recall that if the frame moves with a velocity u, then v_f becomes $v_f + u$ in the new frame, and v_i becomes $v_i + u$ in the new frame.
4. Show that Newton's second law is not valid in a reference frame moving past the laboratory frame of the previous problem with a constant acceleration. Recall that if the frame accelerates with acceleration k, then v_i is still v_i in the new frame at the start but v_f becomes $v_f + k\Delta t$ in the new accelerating frame at the end.
5. A 2000 kg car moving with a speed of 20 m/s collides with and sticks to a 1500 kg car at rest at a stop sign. Show that momentum is conserved in a reference frame moving with a speed of 10 m/s in the direction of the moving car. Recall that in this new reference frame the 2000 kg car is now moving 10 m/s and the stationary car is now moving -10 m/s. For momentum to be conserved in both frames, the final velocity of the two cars stuck together in the new frame should also be 10 m/s less than in the original frame.

Part 2: Time Dilation

6. Two identically constructed clocks are synchronized. One is put in orbit around Earth while the other remains on Earth. Which clock runs slower? When the moving clock returns to Earth, will the two clocks still be synchronized?
7. Two lasers situated on a moving spacecraft are triggered simultaneously. An observer on the spacecraft claims to see the pulses of light simultaneously. What condition is necessary in order that a stationary observer agrees that the two pulses are emitted simultaneously?
8. Suppose astronauts were paid according to the time spent travelling in space. After a long voyage at a speed near that of light, a crew of astronauts return and open their pay envelopes. What will their reaction be?
9. Imagine an astronaut on a trip to Sirius, which lies 8 light years from Earth. Upon arrival at Sirius the astronaut finds that the trip lasted 6 years. If the trip was made at a constant speed of 0.8c, how can the 8 light year distance be reconciled with the 6 year duration?
10. The average lifetime of a pi meson in its own frame of reference is 2.6×10^{-8} s. If the meson moves with a speed of 0.95c, what is its mean lifetime as measured by an observer on Earth?
11. If astronauts could travel at $v = 0.95c$, we on Earth would say it takes (4.2/0.95) = 4.4 years to reach Alpha Centauri, 4.2 lightyears away. The astronauts disagree. How much time passes on the astronauts' clocks?
12. With what speed will a clock have to be travelling in order to run at a rate that is one half the rate when the clock is at rest? (Hint: $t_0/t = 1/2$)

13. An atomic clock is placed in a jet airplane. The clock measures a time interval of 3600 s when the jet moves at a speed of 300 m/s. What corresponding time interval does an identical clock held by an observer on the ground measure? (Hint: For $v/c \ll 1$, note that $\gamma \approx 1 + v^2/2c^2$.)

Part 3: Length Contraction

14. A spaceship in the shape of a sphere moves past an observer on Earth with a speed of $0.5c$. What shape will the observer see as the spaceship moves past?
15. An astronaut moves away from Earth at a speed close to the speed of light. If an observer on Earth could make measurements of the astronaut's size and pulse rate, what changes (if any) would he or she measure? Would the astronaut measure any changes?
16. A spacecraft moves at a speed of $0.9c$. If its length is L_0 when measured from inside the spacecraft, what is its length measured by a ground observer?
17. An astronaut is travelling in a spaceship at 0.8c and flies over a large rectangular warehouse which measures 80 m long by 60 m wide on earth. If the ship is flying in the direction of the length of the building, what dimensions will the astronaut observe for the warehouse?
18. See question 11. What is the distance to Alpha Centauri, as measured by the astronauts?
19. How fast must a metre stick be moving if it is observed to shrink to 0.5 m?
20. How fast would a motorist have to be going to make a red light appear green? ($\lambda_{red} = 650$ nm, $\lambda_{green} = 550$ nm) To compute this, use the correct relativistic formula for the Doppler shift: $\frac{\Delta\lambda}{\lambda} + 1 = \sqrt{\frac{c-v}{c+v}}$ where v is the approach velocity and λ is the source wavelength.
21. Determine the velocity of recession of the quasar 3C9 if its redshift $\Delta\lambda/\lambda = 2$. Use the relativistic formula for the Doppler shift: $\frac{\Delta\lambda}{\lambda} + 1 = \sqrt{\frac{c+u}{c-u}}$ where u is the recessional velocity and $\Delta\lambda$ is the wavelength shift.

Part 4: Energy Mass Equivalence

22. Give a physical argument which shows that it is impossible to accelerate an object of mass m to the speed of light, even with a continuous force acting on it.
23. What happens to the density of an object as its speed increases?
24. Consider the incorrect statement, "Matter can neither be created nor destroyed." How would you correct this statement in view of the special theory of relativity?
25. How is it possible that photons of light with zero mass, have momentum?
26. Does the famous equation $E = mc^2$ take into account all of the energy of an object? If not, what energy is not included?
27. A proton moves with the speed of $0.95c$. Calculate its (a) rest energy, (b) total energy, and (c) kinetic energy.
28. Find the speed of a particle whose total energy is twice that of its rest energy.
29. Electrons are accelerated to an energy of 2×10^{10} eV in the 3 km long Stanford Linear Accelerator. (a) What is the γ factor for the electrons? (b) What is the speed of the 20 GeV electrons?
30. A spaceship of mass 10^6 kg is to be accelerated to $0.6c$. (a) How much energy does this require? (b) How many kilograms of matter and antimatter (in equal proportions) will it take to provide this much energy?